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 **Drag Coefficient(Rev.3.4)** 08 April 2016

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Introduction

The coefficient of aerodynamic drag is a very important parameter having a major effect on the performance of small rockets. In fact, it’s influence on performance is greatest for the smallest rockets due to square-cube effects. Based on physics, there are four sources of drag:

1. Skin friction
2. Wave drag
3. Base drag
4. Parasite drag

Skin friction is what it sounds like, the effect of viscous boundary layer friction between the air and rocket body. Wave drag is an unavoidable consequence of supersonic flow. Base drag is associated with a blunt base behind a body or airfoil. Parasite drag arises from all the little things like uneven joints, riding lugs, etc. Drag due to lift, or induced drag, is not addressed here because sounding rockets fly at very nearly zero lift. Each of these is discussed in its own section below. Finally, the reader is strongly urged to buy him/her self a copy of Dr. Hoerner’s most wonderful book used as ref. 1 in this report.

 The material contained here tends to not generate a smooth curve over a wide range of Mach numbers. This is especially true when transonic flow appears. In such cases, the customary path followed by an analyst is to use his analytical tools whenever possible, and supplement these with a favorite set of french curves. Although it’s not exact, in a practical sense, it works.

Nomenclature

\_\_\_\_\_\_\_\_\_Mnemonic\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_Definition\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

  Wetted area of body,

  Area of one side of one fin panel,

  Number of fin panels,

  Body radius at body station x,

  Aerodynamic reference area,

  Base area of the body,

  Rocket motor nozzle exit area,

  Average Skin friction coefficient,

  Gas constant for air,

  Reynolds number ,

\_\_\_\_\_\_\_\_\_Mnemonic\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_Definition\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

  Reynolds number at boundary layer natural transition,

  Speed of sound in the atmosphere,

  Flight speed,

  Ratio of specific heats of air ,

  Atmospheric mass density,

  Atmospheric viscosity,

  Body overall length,

  Characteristic dimension of the problem,

  Distance from stagnation point to transition,

  Free stream Mach number,

  Ratio of specific heats for air = 1.4,

  Absolute atmospheric temperature,

 $C\_{D}$ Drag coefficient$ =\frac{Drag Force}{½ρU^{2}S\_{ref}}$

 $C\_{PT}$ Stagnation point pressure coefficient

  Base drag coefficient,

  Skin friction drag coefficient,

  Base pressure coefficient,

  Fin plate thickness,

  Fin exposed semispan,

 p Local pressure, and

 p∞ Free stream atmospheric pressure.

Skin Friction Drag

 For the majority of small hobby rockets, skin friction is the dominant source of drag. The tricky part of estimating skin friction drag arises from the fact that the surface boundary layer can take on two forms, laminar and turbulent. Laminar boundary layers have a smooth orderly character whereas turbulent boundary layers are highly disorganized and chaotic. For both types, skin friction depends strongly on Reynolds number.

  (1a)

Here,  is a location of the body and is a flight condition. But,  is an atmospheric property

  (1b)

Thus, to estimate a flight Reynolds number, one obtains the Mach number, the pressure and the absolute temperature from a trajectory simulation or from flight data, and uses these in eq's. (1a) and (1b).

Now, refer to Fig. 1, Chapter II of ref. 1. First, note that it doesn’t matter much whether the surface is flat or cylindrical. Next, consider the Blasius laminar friction coefficient given by

  (2)

The Schoenherr model for incompressible turbulent boundary layers is

  (3a)

A simpler model that is almost as accurate is

  (3b)

We will use eq.(3b) instead of eq. (3a).

 There has probably been more written about transition from the laminar to the turbulent state than on almost any other topic in fluid mechanics. We are more than a century after scientists knew that this was an important problem, but still there is no definitive answer. Here, we adopt the process commonly used industrially. Natural transition is assumed to happen instantaneously at a transition Reynolds number. For typical aircraft/rocket bodies of revolution at speeds less than hypersonic, take

 , (4a)

and for fins take

 . (4b)

 Of course, it is always possible to artificially induce transition by placing some surface roughness, or a boundary layer trip, on the body where transition is desired. DRAG COEFFICIENT3.xls provides for this via an input trip location. Body boundary layer transition then is assumed to occur at the trip site or naturally, whichever is closest to the nose tip.

The reason for the difference in natural transition Reynolds numbers is the boundary layer on the fore body is being continuously thinned as body radius increases moving aft, and therefore is stabilized, as it moves aft. If the fin leading edges are not smoothly rounded to a good approximation to a hemicylindrical contour, boundary layer transition can occur at the low value given by eq. (4b) above. Note that the transition Reynolds number can be shown to make a significant difference in the estimated drag. Since the Reynolds number where the laminar and turbulent skin friction curves cross is about 10,000, it follows that is much larger than the laminar.

 How might this be used to estimate the skin friction drag of a rocket? First, note that for slender bodies, at transition, and down stream, the turbulent skin friction

  ( 5)

When the body is decomposed into elements, and the wetted area found by integration, a plot like Fig. 1 below can be constructed:



Figure 1 Typical Body Wetted Area

A detailed graph of wetted area is not needed, only a few points need be done carefully. So, a conical body element, for instance, can be plotted as a straight line on the wetted area curve even though it's truly parabolic. Just make sure the end points accurate.

 Next, find the maximum length  of laminar boundary layer:

  (6)

If  then the entire area will have a laminar boundary layer, and Eq. (2) can be used to estimate. Otherwise, use Eq. (2) up to the transition point given by Eq. (6). Downstream of the transition point, the boundary layer will be turbulent. The area of laminar boundary requires the  curve be interpolated to find the wetted area ahead of body station , . Then, the remaining turbulent wetted area is



 , (7)

whereis based on. The third term is needed to eliminate the double bookkeeping over the forward (laminar) part of the surface.

 Fin drag can be similarly estimated, but with two slight differences. First, keep in mind that both sides of a fin are subject to skin friction drag. Second, when estimating L and W we assume that all areal elements have the same W, but not the same L. Given a fin area and exposed semispan W find the L that matches the wetted are. The is estimated as for a body, and the skin friction drag is

 . (8)

 Next, consider the effects of compressibility. See page 15-9 and 17-4 of ref. 1. The compressible boundary layer, both flavors, is known to depend on heat transfer, and, if applicable, on ablation. Reference 1 suggests for modest heating, the laminar skin estimate friction need not be altered, and the turbulent boundary layer skin friction as given by Eq. (3) above be reduced according to:

  (9)

assuming an adiabatic wall (probably a good approximation in most practical cases). This has been used in the spread sheet called DRAG COEFFICIENT3.xls to generate skin friction and base drag coefficients. For the body and fins mentioned above.



Figure 2 Typical Skin Friction Breakout

Wave Drag

 Wave drag is associated with the presence of shock waves in the flow around a body. Although these are seen locally at high subsonic Mach numbers, easy classical analyses are not possible until the Mach number is high enough for well-developed supersonic flow. For the wave drag of bodies of revolution, use the techniques of Ref. (3). For the very blunt parts of a rocket, e.g., a hemispherical nose cap on the body, or hemicylindrical fin leading edges, the Modified Newtonian theory of Ref. (4) can be used.

 Consider next the ogive-cylinder shape discussed as an example in the Skin Friction Drag section. The Ref.(3) model has been coded into LOW SUPERSONIC BARROWMAN EQUATIONS3.3.xls. This code has been used to prepare the top plot shown below. The Ref. (4) model has been coded into NEWTON.xls, and used to evaluate the fin leading edge wave drag. The stagnation pressure coefficient is documented later in the Riding Lug section of this memo. It is implemented as a VB function in DRAG COEFFICIENT3. The lower plot, obtained from the spread sheet NEWTON.xls, is the Newtonian wave drag for 4 fins 3/16” thick swept at 45o. Each fin has a hemicylindrical leading edge 0.707 feet long. Note that the fin wave drag is zero when the leading edges are subsonic.



Figure 3. Typical Ogive Wave Drag



Figure 4. Typical Fin Leading Edge Wave Drag

Base Drag

 Reference 1, p 3-19, provides an estimate on incompressible pressure coefficient:

  (10)

Here is the fore body (not including fins) skin friction drag as found above. The effects of compressibility are interesting as shown on p.16-4 of ref. 1. For well-developed supersonic flow, the base drag is approximately 65% of that for a vacuum:

  (11)

For transonic flow, the base drag is independent of Mach number:

  (12)

That is, the transonic base drag is the incompressible base drag given by Eq. (10) above plus the increment given by Eq. (12). The reference point for Eq. (12) is below the fore body critical Mach number, typically about 0.8, or so. This horizontal drag curve is extended in Mach number until it intersects the quasi-vacuum curve given by Eq. (11) above. The maximum Mach number  for the constant base drag coefficient is found to be

  (13)

Finally, the limited data in Ref. (1) for power-on base drag suggest it’s a very complex phenomenon significantly dependent on the nozzle exit pressure. The reader is directed to Ref. 1, p.20-16 which indicates the power-on base drag (based on “unpowered” base area) for well-developed nozzle flow could be 1.5 x that for the power-off case. Physically, the propulsive jet entrains and expels air in the base region thus reducing the base pressure. Absent a better understanding, it’s recommended that this result with the power-off drag be applied to the annulus outside the nozzle. That is,

  (14)

For the example body with a 4” diameter nozzle, power-on base drag is

.

The reader is cautioned that this be a significant contributor to power-on drag. The power-on base drag in this example is 83% of the power-off base drag!

 An example (same body as before) of the power-off base drag is shown below:



Figure 6. Sea Level Base Drag

 Flat plate fin base drag at subsonic speeds can be found from Ref. 1, p. 3-21 to be given by

  (15)

Here  is the skin friction drag of one side of one panel as per Eq. (7) above.

Fin Airfoil Pressure Drag

 The crude airfoil pressure/wave drag model incorporated in DRAG COEFFICIENT3.xls assumes that leading edges are either blunt or sharp, and that trailing edges are also either blunt or sharp. The truth table below shows how the airfoil pressure drag is modeled depending on the airfoil shape:

|  |  |  |  |
| --- | --- | --- | --- |
| Leading Edge | Trailing Edge | Leading Edge Drag Coefficient | Trailing Edge Drag Coefficient |
| Sharp | Sharp | 0 | 0 |
| Blunt | Sharp | 0 if M < 1,Newtonian if M >1 | 0 |
| Blunt | Blunt | Newtonian | Eq.(14) |
| Sharp | Blunt | 0 | Eq.(14) |

This model neglects wave drag of slender airfoil shapes. The Newtonian drag is based of the Newtonian reference. The blunt leading edge-sharp trailing edge model has no subsonic pressure drag due to d'Alembert's paradox.

Riding Lugs

 Two riding lugs are needed to hold a rocket on its launch rail. In appearance they consist of three concentric cylinders as sketched below:

 The rail edges slide between the two larger

 cylinders. The whole assembly is screwed to the

 rocket structure below with a round screw head

 standing above the upper larger cylinder. According

 to ref.(1), section 5, the incompressible drag of such

 a riding lug is about

 

 based on projected frontal area. Then, the

 approximate drag area of a lug is just

 

2 R

h

 Figure 7. Riding Lug Geometry

 A compressible lug drag can be developed assuming the Ref. (4) Newtonian pressure distribution applies. This is strictly accurate only for supersonic free stream Mach numbers, but can be finagled for subsonic free stream Mach numbers. The stagnation pressure coefficient is

  (16)

The pressure ratio above depends on Mach number:

 for M ≤1 (17)

 for M ≥1 (18)

These relations can be used in the cylindrical Newtonian model, Ref. (4) to find the drag area:

  (19)

This is plotted below:



Figure 8. Riding Lug drag Coefficient

If one wished to fudge things to match the Hoerner data, we could take

  (20)

Flat Face Aerodynamics

 Flat faced shapes are very commonly encountered, especially in conjunction with large angles of attack. First, Ref. 5 provides a comparison with experiment over a wide range of supersonic Mach numbers. The wind tunnel data therein all lie within 4% of Eq. (21). The streamline impacting the nose stagnation point passes through a normal shock before being isentropically compressed to zero speed. Equation (100) of ref. (1) is the Rayleigh pitot formula. It states that

  (100)

The stagnation pressure coefficient is, by definition,

  (21)

This equation is plotted below in Fig. 1:



Figure 9. Supersonic Stagnation Point Pressure Coefficient

Flat Faced Nose Aerodynamics

 Flat faced shapes are very commonly encountered, especially in conjunction with large angles of attack. First, Ref. 8 provides a comparison with experiment over a wide range of supersonic Mach numbers. The wind tunnel data therein all lie within 4% of Eq. (21). The methods of Ref. (2), Chapter 15, suggest a subsonic model of the form

$ C\_{D}=0.85(1+ ¼\left(M^{2}+M^{4}\right))$ (22)

The result of Eq’s. (21) and (22) plotted together are shown below:

Figure 10. Flat Faced Nose Drag Coefficient

Parasite Drag

 Parasite drag is often considered to be the dogs and cats of the drag world. It encompasses all the drag sources too small individually to merit formal analyses like those outlined above. Examples include skin joints, surface roughesses and RF antennas. Many, if not all, of these could be analyzed with the same methods described above, but the additional analysis is often considered a poor use of engineering resources. As a result, one frequently accounts for parasite drag by adding an additional 5%, or so, to the total drag obtained from formal itemized analysis.

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